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Evaluating ASR-9 Radar Azimuth Error Models Through Analysis of Targets of Opportunity Data

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I. INTRODUCTION

Over the next decade the Federal Aviation Administration (FAA) is planning a wide ranging transformation of the national air transportation system, known as NextGen. With NextGen the FAA will move away from single sensor ground based surveillance systems towards a satellite based surveillance system known as Automatic Dependent Surveillance Broadcast (ADS-B) [1]. Currently the FAA Order 7110.65 detailing the surveillance requirements to support separation services assumes that surveillance is provided by single sensor radar technology [2]. To extend separation services to other surveillance systems, their performance must be proven equivalent to or better than the existing radar system performance. This comparison is often conducted using radar error models to characterize radar performance.

Over the last decade, several studies sponsored by the FAA have employed a single Gaussian distribution to model the azimuth jitter, error in the measurement of the aircraft's azimuthal position relative to the radar beam, of a monopulse secondary surveillance radar (MSSR) [3]. However, as part of a recent study in Europe the Radio Technical Commission for Aeronautics (RTCA) Requirements Focus Group (RFG) developed a new radar error model for MSSR azimuth jitter that focused more on low probability errors, the tails of the error distribution. They determined in their analysis that a single Gaussian distribution was too narrow and did not adequately model low probability MSSR azimuth jitter. The RFG instead chose to employ a double Gaussian distribution with wider tails to model MSSR azimuth jitter [4].

In this paper we evaluate the suitability of the two radar error models for use in the establishment of separation standards by comparing them to a large collection of radar errors. The radar errors are calculated using a technique developed at MIT Lincoln Laboratory to estimate radar azimuth jitter from large, widely distributed sets of radar data. For our analysis we show results from over two million azimuth error samples collected from several ASR-9 Mode S MSSR within the national airspace system (NAS). We then best fit single

and double Gaussian distributions to the sampled error and compare them to the established error models. Finally, we fit a third distribution type, a Gaussian Laplace (or Gaussian-exponential) distribution to the error distribution, and compare it to the other models through various metrics.

II. BACKGROUND

A. Radar Azimuth Errors

Radar azimuth errors consist of a combination of two different errors: azimuth measurement error and radar azimuth bias. Azimuth measurement error, or azimuth jitter, is the error in the measurement of the aircraft's azimuthal position relative to the radar beam. Radar azimuth bias is a systematic error caused by a misalignment of the radar with true north. In the existing pre-ADS-B single sensor radar system, radar azimuth bias has minimal effect on surveillance performance with respect to separation services. Single sensor radar systems apply a sensor's azimuth bias equally to all aircraft and the bias is in effect canceled out in any separation measurements. Consequently, when using single sensor radar as a reference system, the system and its corresponding radar error models are usually assumed to have no azimuth bias. All radar errors and error models mentioned in this paper refer solely to the azimuth measurement error and do not account for any radar azimuth bias.

B. Single Gaussian Error Model

The traditional single Gaussian model for radar azimuth jitter is a normal distribution with zero-mean. Its probability density function (PDF) is defined as follows:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} \quad (1)$$

The standard deviation, σ , for this MSSR azimuth error model is accepted to be 0.068° (0.8 Azimuth Change Pulse) [5].

C. Double Gaussian Error Model

The RFG determined that the single Gaussian distribution underestimated the low probability MSSR azimuth errors. They developed a new error model using a double Gaussian distribution, the weighted sum of two normal distributions. The model consists of a heavily weighted and narrower normal

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distribution, similar to the single Gaussian error model, and a lightly weighted and wider normal distribution intended to characterize larger low probability azimuth errors. The model is mathematically defined as follows:

$$f(x) = (1 - \alpha) \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{x^2}{2\sigma_1^2}} + \alpha \frac{1}{\sqrt{2\pi\sigma_2^2}} e^{-\frac{|x|}{2\sigma_2^2}} \quad (2)$$

where the accepted value of the weighting factor α is .05, and the standard deviations σ_1 and σ_2 are $.054^\circ$ and 0.27° respectively [4].

D. Gaussian Laplace Distribution

We evaluated a third distribution type, a Gaussian Laplace, as a potential model for MSSR azimuth error. Similar to the double Gaussian error model, the Gaussian Laplace is a weighted sum of two distributions; a narrow normal distribution to model typical smaller azimuth errors and a wider distribution, in this case a Laplace or double exponential distribution, to model larger low probability azimuth errors. The functional form of the Gaussian Laplace distribution is defined as:

$$f(x) = (1 - \alpha) \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} + \alpha \frac{1}{2\lambda} e^{-\frac{|x|}{\lambda}} \quad (3)$$

where α is the weighting factor, σ is the standard deviation of the normal distribution, and λ is the scaling parameter of the Laplace distribution.

E. Azimuth Error Estimation Technique

The radar error estimation technique developed at MIT Lincoln Laboratory accurately estimates radar azimuth jitter using raw time-stamped range/azimuth radar reports from aircraft flying through the NAS, known as targets of opportunity (TOO), providing an efficient solution for analyzing large quantities of radar data. The method filters TOO data so that it contains only aircraft flying straight and level at a near constant velocity, and then uses this a priori knowledge of the aircraft behavior to accurately estimate the true aircraft position and radar error.

The estimation technique involves a multi-step process. First, tracks of raw secondary reports from a single sensor recorded in radar coordinates (azimuth, slant range, and altitude) are projected onto a stereographic plane and stored in Cartesian coordinates (x, y) relative to the sensor. Each track is then passed through a filter that calculates the smoothed heading and velocity of the aircraft and extracts periods of straight and level flight at a near constant velocity. The straight and level tracks are then fed to the estimation algorithm. The algorithm estimates the true trajectory of the aircraft as the line that best fits the track in a least-squares sense. The algorithm estimates the true position of the aircraft at the time of each radar measurement by finding the set of points (x'_i, y'_i) on the least-squares line that minimize the summed distance between the points and the measurements (x_i, y_i) , under the constraint of the aircraft flying at a constant groundspeed v .

TABLE I
ASR-9 SENSORS

Site Location	Site ID
Baltimore, MD	BWI
Chicago, IL	ORD
Boston, MA	BOS
Los Angeles, CA	LAX
Manchester, NH	MHT
New York, NY	JFK
Newark, NJ	EWR

More precisely, we are in search of the following set:

$$\{(x'_i, y'_i) | \min_{x_i, y_i} \sum_{i=1}^m d_i \frac{\sqrt{(x'_{i+1} - x'_i)^2 + (y'_{i+1} - y'_i)^2}}{t'_{i+1} - t'_i} = v\} \quad (4)$$

where d_i is the distance between the points (x_i, y_i) and (x'_i, y'_i) , and t'_i and t'_{i+1} are the time stamps of consecutive radar reports. The estimated true positions are then converted back into radar coordinates and the azimuth jitter is calculated for each measurement.

A detailed explanation and validation of the estimation technique is found in [6].

III. ANALYSIS

A. Data Collection

Radar data was supplied by the 84th Radar Evaluation Squadron (RADES). The data set consisted of over 7 million radar reinforced beacon reports collected from 8 different ASR-9 sensors over a period of 14 days from February 28th to March 13th, 2009. Table 1 lists the ASR-9 sensors chosen for the analysis. The radar reports consisted of range, azimuth and pressure altitude measurements. Prior to projection onto a stereographic plane the pressure altitude measurements needed to be corrected using local weather data. Aircraft altitude is reported as Mode C pressure altitude from sea level on an average day, barometric pressure of 29.92 mm Hg. As local barometric pressure deviates from this standard, Mode C altitude reports become inaccurate. To ensure accurate position projections Mode C altitude reports were corrected to the true altitude using the local hourly barometric pressure and temperature. Each altitude corrected radar report was then projected onto a stereographic plane tangential to its sensor and stored in a database in both radar and Cartesian coordinates.

B. Model Fitting Technique

Three different distributions were fitted to the estimated azimuth errors and evaluated for accuracy. Because only radar azimuth jitter was estimated, the error distribution is zero mean, and therefore the fitted distributions are assumed to be zero mean as well. The functional form of the three different distributions and the unknown parameters that need to be estimated, α , σ_x , and λ , are listed in Eq. (1-3).

A sample PDF of the estimated azimuth error was created by counting the frequency of occurrence of azimuth errors

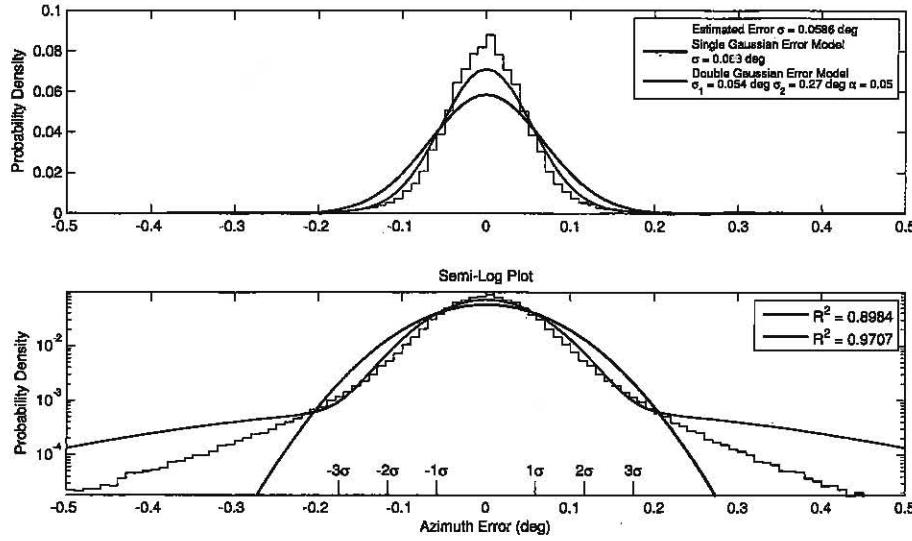


Fig. 1. Estimated Azimuth Error vs Established Error Models

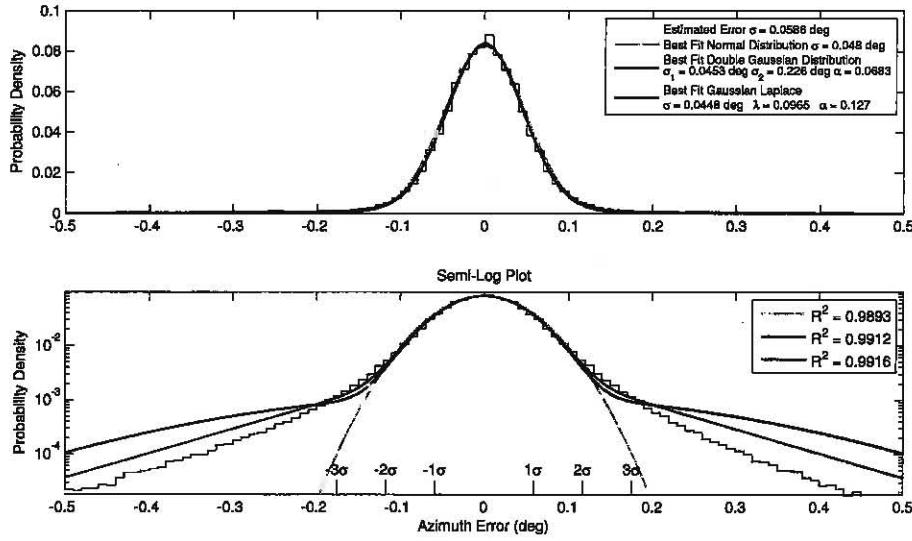


Fig. 2. Estimated Azimuth Error vs Best Fit Error Models

in bins of size 0.01° ranging from -0.5° to 0.5° . Then the distribution was normalized by dividing the count in each bin by the total number of samples. The best fit parameters for each distribution type were determined from a least-squares minimization of the sum of the differences between the model PDF value and estimated error PDF value at each bin location.

C. Model Evaluation Metrics

Models were evaluated for overall goodness of fit using the coefficient of determination or R^2 value. Additionally the estimated error PDF was broken down into σ bands (i.e. $|0$ to $\sigma|$, $|\sigma$ to $2\sigma|$, etc.) and each model was evaluated for quality of fit in each band using the following metrics:

- Probability difference
 $|\Pr(\text{model PDF}) - \Pr(\text{estimated PDF})|$
- Percent difference
 $\frac{|\Pr(\text{model PDF}) - \Pr(\text{estimated PDF})|}{|\Pr(\text{model PDF}) + \Pr(\text{estimated PDF})|/2} \cdot 100\%$

IV. RESULTS

Figure 1 illustrates the comparison between the estimated error PDF and the two established radar error models. Figure 2 shows the comparison between the estimated error PDF and the best fit single Gaussian, double Gaussian, and Gaussian Laplace distributions. The locations of σ values beyond $|3\sigma|$ were left out for the sake of clarity. The results in each figure are plotted on both linear and semi-log scales, the upper and lower plots respectively. The parameters for each

	1σ	$1\sigma-2\sigma$	$2\sigma-3\sigma$	$3\sigma-4\sigma$	$4\sigma-5\sigma$	$5\sigma-6\sigma$	$6\sigma-7\sigma$	$7\sigma-8\sigma$
Count	5,428,481	1,339,822	213,027	57,338	20,678	9,262	4,466	2,324
Measured Probability	0.6495	0.2160	0.0328	0.0083	0.0030	0.0013	0.0005	0.0004
Double Gaussian Error Model	Model Probability	0.5966	0.2703	0.0380	0.0078	0.0055	0.0043	0.0028
	Difference	0.0529	0.0543	0.0052	0.0005	0.0025	0.0030	0.0021
	% Difference	8.49%	22.32%	14.64%	5.28%	60.17%	107.98%	133.69%
Single Gaussian Error Model	Model Probability	0.6231	0.3277	0.0805	9.4672E-03	5.2808E-04	1.3938E-05	1.7139E-07
	Difference	0.1264	0.1116	0.0477	0.0012	0.0024	0.0013	0.0006
	% Difference	21.55%	41.07%	84.24%	13.41%	138.42%	195.72%	199.87%
Best Fit Single Gaussian	Model Probability	0.6659	0.2346	0.0181	2.6834E-04	9.3523E-07	7.4198E-10	1.2723E-13
	Difference	0.0164	0.0186	0.0167	0.0080	0.0030	0.0013	0.0005
	% Difference	2.49%	8.25%	68.13%	187.86%	199.87%	200.00%	200.00%
Best Fit Double Gaussian	Model Probability	0.6544	0.2119	0.0219	0.0097	0.0073	0.0052	0.0030
	Difference	0.0049	0.0042	0.0109	0.0014	0.0043	0.0039	0.0024
	% Difference	0.76%	1.96%	40.04%	15.85%	84.60%	120.26%	137.47%
Best Fit Gaussian-Laplace	Model Probability	0.6324	0.2154	0.0187	0.0087	0.0072	0.0053	0.0030
	Difference	-0.0156	0.0031	0.0001	0.0004	0.0002	0.0001	0.0000
	% Difference	0.20%	0.20%	10.61%	16.20%	15.37%	15.11%	15.07%

Fig. 3. Model Evaluation Metrics

distribution are shown in the legends of the upper plots while the R^2 goodness-of-fit statistics are shown in the lower plots' legends. The detailed model evaluation metrics for each distribution are shown in Figure 3, where the measured and model probabilities refer to the probability from the specified PDF of an error occurring in each σ band.

The results confirm the conclusion of the RFG study [4] that the single Gaussian error model does not capture the larger low probability MSSR azimuth errors. For work where these low probability MSSR errors are of concern a distribution with wider tails (e.g. double Gaussian or Gaussian Laplace error model) would be more appropriate. This analysis found the best fit Gaussian Laplace error model to be more accurate than the best fit double Gaussian error model, especially at very low probabilities, greater than 4σ from the distribution mean. However, the relative sample sizes at these low probabilities are very small and the difference between the performance of the two models is on the same order as the estimation noise.

In applications where only the nominal performance of the radar is of concern, azimuth errors between -2σ and 2σ of the mean of the error distribution, a single Gaussian error model might be adequate. However, the current established single Gaussian model uses a statistical standard deviation of the azimuth error, 0.068° , as the standard deviation of the single Gaussian distribution which in this analysis was shown to overestimate the nominal azimuth error. A best fit single Gaussian similar to the one presented in this paper would be a more conservative model and might be more appropriate for reference system analysis.

V. CONCLUSION

A large scale analysis was performed on ASR-9 data to evaluate different azimuth error models. The results confirmed that a single Gaussian error model does not capture larger low probability MSSR azimuth errors and that a double Gaussian or Gaussian Laplace distribution would be more appropriate for modeling those errors. However, when modeling nominal radar performance it was found that a single Gaussian distribution is adequate and a best fit Gaussian might be more appropriate for reference system analysis as it would be a more conservative error model.

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